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Statistical Methods of Maximizing the Accuracy of Empirical Models

N. A. Ragrin[a, *] **and D. M. Kurganova**[b, **]

[a] *Yeltsin Kyrgyz–Russian Slavic University, Bishkek, 44 Kyrgyz Republic*

[b] *Razzakov Kyrgyz–Russian State Technical University, Bishkek, 66 Kyrgyz Republic*

**e-mail: n_ragrin@mail.ru*

***e-mail: darina220699@gmail.com*

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Abstract—Methods of statistically assessing the factors affecting the generalized variance are validated. After analysis of the factors' influence on the generalized variance, methods are established for statistically assessing this influence, thereby maximizing the accuracy of empirical models.

Keywords: experiments, factors, variance, criteria, correlation, regression, statistical assessment, accuracy, empirical models

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The goal of experiments is often to derive empirical models ensuring high product quality at low cost. The accuracy of empirical models determines the agreement between the calculation results and the actual experimental data, which largely depends on the methods of analysis.

In the present work, we validate and assess statistical methods of analyzing experimental results, thereby maximizing the accuracy of empirical models.

Statistical methods such as variance analysis, correlation analysis, and regression analysis are only applicable if the distribution of the experimental results has been proven to be normal. For engineering experiments with a range of random interference, the normal distribution is adopted on the basis of the Lyapunov theorem: if a random quantity is the sum of a very large number of mutually independent random quantities, each of which has little effect on the aggregate effect, its distribution will be close to normal.

The normal distribution is completely determined by two parameters: the mean and the variance. Therefore, with the normal distribution, a factor can affect only two parameters: a qualitative characteristic of the experimental data, the generalized variance; and/or a quantitative characteristic, the generalized mean. However, if the experimental method is unchanged, we may assume that the generalized variance is also unchanged. In that case, the statistical methods considered are based on assessing the influence of each factor on the generalized mean.

In engineering research, the variable factors are machining parameters of the product, as a rule. The experimental results—denoted by y_i—are measurements of the characteristics and the cost of the product. The result of an experiment with a single factor is a series of measurements having the following characteristics: the number of measurements, the sample size n; a qualitative characteristic, the sample variance s_i^2; and a quantitative characteristic, the sample mean $\bar{y}_i$.

The generalized variance σ^2 cannot be found from the results of a single experiment. It may be found approximately from the sample variance s^2. The error of this approximation depends on the sample size. As a rule, this error is disregarded when $n > 50$. With only a single factor, this number of measurements is rare. Therefore, the sample variances calculated from the quantitative characteristics of the experimental results are compared. The comparison of sample variances to determine the influence of specific factors on the experimental result is known as variance analysis.

The main question in variance analysis is whether the sample variances being compared may be regarded as estimates of the same generalized variance. If so, then the factor under consideration may be assumed to have no significant influence on the generalized variance and on the accuracy of the experimental method. In that case, the next step is to determine the influence of the factor on the generalized mean. If that influence is found to exist, correlation analysis is required to assess its significance, with the goal of determining whether regression analysis may be used to calculate the regression factors for the empirical formulas that are needed to construct an empirical model.

The following research goals are defined:

(1) to validate methods of assessing the influence of a specific factor on the qualitative characteristic of the experimental research: the generalized variance;

Table 1

S, mm/turn	HRC (y_i)									
$S_1 = 0.10$	13.8	8.9	12.7	13.4	11.6	9.1	11.2	10.7	11.4	12.5
$S_2 = 0.13$	10.3	10.1	9.2	9.0	11.9	9.7	10.8	11.2	10.4	11.4
$S_3 = 0.16$	9.8	10.5	17.6	9.2	11.2	10.4	11.3	12.4	10.9	9.9
$S_4 = 0.20$	10.0	18.5	11.3	11.5	13.7	12.5	12.5	12.1	12.1	11.1
$S_5 = 0.25$	15.7	12.4	24.7	11.9	16.8	13.0	13.3	13.4	13.8	12.9

(2) to validate methods of assessing the influence of the factor on the quantitative characteristic of the experimental research: the generalized mean;

(3) to identify, validate, and assess statistical methods by which the accuracy of empirical models may be maximized.

The experiments used to validate and assess statistical methods by which the accuracy of empirical models may be maximized were itemized in [1].

The factor chosen for study is the supply S—that is, the cutting parameter with the greatest influence on the strength of the machined surface [2]. The experimental result y_i is the hardness HRC of the machined surface. Table 1 presents the values of the factor and the measured hardness of the machined surface.

VALIDATING METHODS FOR ASSESSING THE INFLUENCE OF THE FACTOR ON THE GENERALIZED VARIANCE

Table 1 presents five series of measurements for the hardness HRC (y_i) of the machined surface with different values of the supply S.

To verify that the sample variances belong to the same generalized variance, we compare two sample variances from the series (s_1^2, s_2^2, s_3^2, s_4^2, s_5^2), which have degrees of freedom ($n_1 - 1$, $n_2 - 1$, $n_3 - 1$, $n_4 - 1$, $n_5 - 1$) for each sample size n. We assume that the first sample has generalized variance σ_1^2, while the second has generalized variance σ_2^2. Our hypothesis is that these variances are equal: $\sigma_1^2 = \sigma_2^2$. This hypothesis will be regarded as correct if there is no significant discrepancy between them. The significance will be assessed in terms of the critical points of the Fisher distribution. We judge whether the discrepancy is significant on the basis of the ratio of sample variances s_1^2/s_2^2 according to tables of critical points of the Fisher distribution as a function of the significance level F_{1-p} and the combination of degrees of freedom [3]

$$\frac{1}{F_{1-p}(n_2 - 1,\, n_1 - 1)} \le \frac{s_1^2}{s_2^2} \le F_{1-p}(n_1 - 1,\, n_2 - 1). \quad (1)$$

The larger of the two variances is denoted by s_1^2.

The left side of Eq. (1) is always satisfied, since $s_1^2/s_2^2 \ge 1$. Therefore, if the right side of Eq. (1) holds, the hypothesis that the sample variances are equal is true. If that hypothesis is not confirmed, we conclude that the factor under consideration affects the accuracy of the experimental method. That is impermissible, since it would violate the requirement of unchanging experimental methods.

Note that the statistical criteria may only be used if the distribution of the measurements is normal. To that end, anomalous measurements (outliers) must be eliminated; they are identified by the Grubbs method [4–6].

The use of these criteria is governed by State Standard GOST R ISO 5725-2–2002 [7]: specifically, the Grubbs index G_P is calculated to determine whether the largest value in the series of increasing data x_i (i = 1, 2, ..., p) is an outlier. The formula employed is

$$G_P = (x_P - \bar{x})/s, \quad (2)$$

where $\bar{x}$ is the mean of the measurement results

$$\bar{x} = \frac{1}{p}\sum_{i}^{n} x_i, \quad (3)$$

and s is the standard deviation

$$s = \sqrt{\frac{1}{p-1}\sum_{i}^{n}(x_i - \bar{x})^2}. \quad (4)$$

To check the significance of the lowest measurement, we calculate the test statistic

$$G_1 = (\bar{x} - x_1)/s. \quad (5)$$

If $G_1 \le 5\%$ of the critical Grubbs index, the hypothesis is correct. If the largest measurement is an outlier (more than 1% of the critical Grubbs index), the lowest measurement must be verified, and the largest value must be eliminated. The critical values for the Grubbs index may be found in State Standard GOST R ISO 5725-2–2002 [7]. However, in that table the significance levels are incorrectly stated, as noted in [8]. The values corresponding to levels of 0.5 and 2.5% are assigned to levels of 1 and 5% in the initial source [6]. When using the critical values for the Grubbs index in State Standard GOST R ISO 5725-2–2002 [7], some of the outliers will not be discarded. There-

Table 2

S, mm/turn	G_P	Critical Grubbs index	
		5%	1%
$S_1 = 0.10$	1.376	2.176	2.410
$S_2 = 0.13$	1.255		
$S_3 = 0.16$	2.630		
$S_4 = 0.20$	2.550		
$S_5 = 0.25$	3.790		

Table 3

S, mm/turn	G_P	Critical Grubbs index	
		5%	1%
$S_1 = 0.10$	1.59	2.176	2.410
$S_2 = 0.13$	1.17		
$S_3 = 0.16$	1.485	2.110	2.323
$S_4 = 0.20$	1.78		
$S_5 = 0.25$	0.82		

Table 4

S, mm/turn	G_P	Critical Grubbs index	
		5%	1%
$S_3 = 0.16$	1.86	2.110	2.323
$S_4 = 0.20$	1.745		
$S_5 = 0.25$	1.96		

fore, in the assessments that follow, we adopt 5% of the critical values given for the Grubbs index in [6].

Table 1 presents five series of measurements for the hardness of the machined surface with different values of the supply S. If we assume that the experimental method is unchanged with different values of S, it follows that the generalized variance also remains constant. That provides the basis for assessing the outliers in a single series of hardness measurements with the same supply S.

Table 2 presents the Grubbs indices calculated from Eqs. (2)–(4) for the greatest hardness values in each series of measurements (Table 1) and the critical Grubbs index [6]. The greatest hardness values of the machined surface obtained when S = 0.1 and 0.13 mm/turn are correct. The greatest hardness values of the machined surface obtained when $S = 0.16$, 0.2, and 0.25 mm/turn are outliers. They must be eliminated, and the lowest hardness values in those series must be verified.

Table 3 presents the Grubbs indices calculated from Eqs. (3)–(5) for the lowest hardness values in the first two series (Table 1) and the next three series without outliers, as well as the critical Grubbs indices [6]. The lowest hardness values are correct for all values of the supply S.

Table 4 presents the results of verifying the greatest hardness values in the third, fourth, and fifth series after eliminating the outliers. The greatest hardness values are correct for all values of the supply S.

After eliminating the outliers from Table 1, we obtain Table 5.

If sequential elimination of the outliers is an effective method, the remaining measurements should correspond to a normal distribution. Verification that the distribution of measurements with $15 < n \le 50$ is normal is governed by State Standard GOST R 8.736–201 [9]. The basis for this standard is that, with less than 50 measurements, the conventional Pearson criterion χ^2 and von Mises–Smirnov criterion ω^2 are ineffective or permit excessive error. When $15 < n \le 50$, State Standard GOST R 8.736–201 permits verification that the distribution is normal on the basis of two criteria: B1 and B2 [9].

(1) The formula for criterion B1 is

$$\bar{d} = |x_i - \bar{x}|/ns^*, \tag{6}$$

where

$$s^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}. \tag{7}$$

The measurements correspond to a normal distribution if

$$d_{1-q/2} < \bar{d} \le d_{q/2}, \tag{8}$$

where $d_{1-q/2}$ and $d_{q/2}$ are quantiles of the distribution [9].

Table 5

S, mm/turn	HRC (y_i)									
$S_1 = 0.10$	13.8	8.9	12.7	13.4	11.6	9.1	11.2	10.7	11.4	12.5
$S_2 = 0.13$	10.3	10.1	9.2	9.0	11.9	9.7	10.8	11.2	10.4	11.4
$S_3 = 0.16$	9.8	10.5	9.2	11.2	10.4	11.3	12.4	10.9	9.9	–
$S_4 = 0.20$	10.0	11.3	11.5	13.7	12.5	12.5	12.1	12.1	11.1	–
$S_5 = 0.25$	15.7	12.4	11.9	16.8	13.0	13.3	13.4	13.8	12.9	–

Table 6

S, mm/turn	s_i^2	Variance ratio		Critical points of Fisher distribution	
$S_1 = 0.10$	$s_1^2 = 2.94$	$s_1^2/s_2^2 = 2.2$	$s_2^2/s_4^2 = 1.2$	3.18	3.23
$S_2 = 0.13$	$s_2^2 = 1.33$	$s_1^2/s_3^2 = 3.23$	$s_5^2/s_2^2 = 1.87$	3.23	3.39
$S_3 = 0.16$	$s_3^2 = 0.91$	$s_1^2/s_4^2 = 2.67$	$s_4^2/s_3^2 = 1.21$	3.23	3.44
$S_4 = 0.20$	$s_4^2 = 1.1$	$s_1^2/s_5^2 = 1.18$	$s_5^2/s_3^2 = 2.73$	3.23	3.44
$S_5 = 0.25$	$s_5^2 = 2.49$	$s_2^2/s_3^2 = 1.46$	$s_5^2/s_4^2 = 2.26$	3.23	3.44

(2) In the case of criterion B2, we assume that the measurements conform to a normal distribution if no more than m differences $|x_i - \bar{x}|$ exceed the value $Z_{p/2}s$, where $Z_{p/2}$ is the upper quantile of the distribution normalized to the Laplacian that corresponds to probability $P/2$. The probabilities P and m are determined for the selected significance level and number of measurements from Table B.2 in [9]. The dependence of $Z_{p/2}$ on P is given in Table B.3 of [9]. Violation of at least one criterion indicates that the measurements in the group are not normally distributed.

Verification of the measurements on the basis of criterion B1 (Table 5) shows that, for the value $\bar{d} = 0.8202$ calculated from Eqs. (6) and (7), the quantile $d_{q/2} = 0.8481$ (5% significance level) is the least in Table B.1 [9]; the quantile $d_{1-q/2} = 0.7518$ (95% significance level) is the greatest. In other words, Eq. (8) is satisfied.

Verification of the measurements on the basis of criterion B2 (Table 5) shows that no more than two differences $\left|HRC_i - \overline{HRC}\right|$ (where $\overline{HRC} = 11.5936$) exceed the value $Z_{p/2}s = 3.94$ (Tables B.2 and B.3 in [9]). In other words, criterion B2 is satisfied.

Criteria B1 and B2 confirm the normal distribution of the measurements. Consequently, variance analysis may be used to assess the influence of the selected factor (the supply S) on the qualitative characteristic of the experimental data, the generalized variance. Table 6 presents the results of variance analysis.

We assess the significance on the basis of the critical points of the Fisher distribution at a significance level of 0.05 (Table 6 in [10]). We see that there is no relationship between the factor (the supply S) and the sample variances. All of the sample variances without exception belong to the same generalized variance. Within the limits considered, S has no influence on the sample variances and hence no influence on the experimental accuracy. We may now proceed to the next step: assessing its influence on the quantitative characteristic of the results, the generalized mean.

INCREASING THE ACCURACY OF MODELS: ASSESSING THE INFLUENCE OF THE FACTOR ON THE GENERALIZED MEAN

To determine whether the sample means being compared (say, $\bar{y}_1$ and $\bar{y}_2$) correspond to the same generalized mean, the initial hypothesis is that the generalized means are equal: $\alpha_1 = \alpha_2$. If this hypothesis is true, then

$$|\bar{y}_1 - \bar{y}_2| < u_{1-p/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \tag{9}$$

where $u_{1-p/2}$ is a quantile of the normal distribution.

The quantiles of the normal distribution are simple and reliable but they require knowledge of the generalized variances σ_1^2 and σ_2^2. That is impossible when working with small samples. If the sample variances are unknown, the only option is to use the sample variances s_1^2 and s_2^2. That entails resorting to Student's test. According to the Fisher test, the sample variances s_1^2, s_2^2, s_3^2, s_4^2, s_5^2 correspond to the same generalized variance σ_1^2. In that case, the generalized variance may be assessed in terms of a weighted mean variance such as

$$s_{1.2}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

As already shown, this set of measurements corresponds to a normal distribution. Then Eq. (9) may be written in the form

$$|\bar{y}_1 - \bar{y}_2| < t_{1-p/2}\sqrt{s_{1.2}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}, \tag{10}$$

where $t_{1-p/2}$ is a quantile of the Student distribution (Table III in [3]).

From Eq. (10), we may determine a criterion for assessing whether the sample means $\bar{y}_1$ and $\bar{y}_2$ correspond to a single generalized mean a

$$t_{1.2} = \frac{|\bar{y}_1 - \bar{y}_2|}{\sqrt{s_{1.2}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}.$$

Table 7

S, mm/turn	$\overline{HRC}(\bar{y})$	t	$t_{1-p/2}$
$S_1 = 0.10$ $S_2 = 0.13$	$\overline{HRC}_1(\bar{y}_1) = 11.53$ $\overline{HRC}_2(\bar{y}_2) = 10.4$	$t_{1.2} = 1.73$	2.10
$S_3 = 0.16$ $S_4 = 0.20$	$\overline{HRC}_3(\bar{y}_3) = 10.62$ $\overline{HRC}_4(\bar{y}_4) = 11.87$	$t_{3.4} = 2.64$	2.12
$S_5 = 0.25$	$\overline{HRC}_5(\bar{y}_5) = 13.69$	$t_{4.5} = 2.88$	2.12

If $t_{1.2} \geq t_{1-p/2}$, the hypothesis that the two generalized means are equal ($a_1 = a_2$) must be rejected. In other words, the sample means $\bar{y}_1$ and $\bar{y}_2$ do not correspond to a single generalized mean.

Table 7 presents the assessment of whether the sample means correspond to a single generalized mean. We see that the sample means $\bar{y}_1$ and $\bar{y}_2$ do correspond to a single generalized mean.

The factor of interest—the supply S—has no influence on the quantitative characteristics of the experimental results, since $t_{1.2} < t_{1-p/2}$. As we see, the results are the same when $S_1 = 0.1$ mm/turn and $S_2 = 0.13$ mm/turn. The sample means $\bar{y}_3$, $\bar{y}_4$, and $\bar{y}_5$ do not correspond to a single generalized mean. The supply S has a significant effect on the quantitative characteristics of the experimental results (comparing the values for S_3, S_4, and S_5. Hence, $t_{3.4} > t_{1-p/2}$ and $t_{4.5} > t_{1-p/2}$. The positive finding $t > t_{1-p/2}$ only indicates that the supply affects the corresponding experimental indices. To assess the significance of this influence entails correlation analysis and determination of the magnitude and significance of the Pearson linear correlation coefficients between the values of the supply S and the corresponding sample means. However, unambiguous assessment of the influence of the supply S on the generalized mean is possible by determining the values of S corresponding to the maximum effect in correlation analysis.

CORRELATION ANALYSIS OF THE RESULTS

Methods of correlation analysis were validated in [11]. In addition, the presence and significance of the relation between nonrandom factors and the random results of empirical engineering research were determined, and a formula for the Pearson linear correlation coefficients was given

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}, \tag{11}$$

where x is the supply S, which is not random.

Given that the quantitative characteristic of the experimental result y_i is the mean of a single series of measurements, we may write Eq. (11) in the form

$$r = \frac{\sum (x_i - \bar{x})(\bar{y}_i - \bar{\bar{y}})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (\bar{y}_i - \bar{\bar{y}})^2}}. \tag{12}$$

Table 8 presents the data required to calculate the Pearson linear correlation coefficient from Eq. (12).

On the basis of Eq. (12) and Table 8, the Pearson coefficient between the supply and the corresponding values of the sample means is $r = 0.999$. That corresponds to the quantile of the distribution of the sample correlation coefficient $r_{1-p/2}$ at the significance level $p = 0.001$ with the maximum value for the given number of degrees of freedom (Table XII in [3]). That confirms the maximum accuracy of the results of correlation analysis.

REGRESSION ANALYSIS OF THE RESULTS

Given the high significance of the Pearson linear correlation coefficient, the relation between the hardness of the machined surface and the supply (Table 8) may be approximated by a linear dependence $y = \alpha + bx$. On the basis of mathematical log-linear transformation and the least squares principle, the regression factors may be approximated by nonlinear expressions—power laws $y = \alpha x^b$ and exponential formulas $y = \alpha b^x$—with subsequent conversion to linear

Table 8

S, mm/turn (x_i)	$\overline{HRC}$	$x_i - \bar{x}$	$\bar{y}_i - \bar{\bar{y}}$	$(x_i - \bar{x})^2$	$(\bar{y}_i - \bar{\bar{y}})^2$	$(x_i - \bar{x})(\bar{y}_i - \bar{\bar{y}})$
0.16	10.62	–0.043	–1.44	0.001849	2.0736	0.06192
0.2	11.87	–0.003	–0.19	0.000009	0.0361	0.00057
0.25	13.69	0.047	1.63	0.002209	2.6569	0.07661
$\mathbf{\bar{x} = 0.203}$	$\mathbf{\bar{\bar{y}} = 12.06}$	–	–	$\sum (x_i - \bar{x})^2 = \mathbf{0.004067}$	$\sum (\bar{y}_i - \bar{\bar{y}})^2 = \mathbf{4.7666}$	$\sum (x_i - \bar{x})(\bar{y}_i - \bar{\bar{y}}) = \mathbf{0.1391}$

Table 9

x_i (S, mm/turn)	$\overline{y}_i$ ($\overline{HRC}$)	x_i^2	$\overline{y}_i^2$	$x_i\overline{y}_i$	$(x_i+\overline{y}_i)^2$
0.16	10.62	0.0256	112.7844	1.6992	116.2084
0.20	11.87	0.04	140.8969	2.374	145.6849
0.25	13.69	0.0625	187.4161	3.4225	194.3236
Σ = 0.61	**36.18**	**0.1281**	**441.0974**	**7.4957**	**456.2169**

Table 10

$\ln x_i$ ($\ln S$)	$\ln\overline{y}_i$ ($\ln\overline{HRC}$)	$\ln x_i^2$	$\ln\overline{y}_i^2$	$\ln x_i\ln\overline{y}_i$	$(\ln x_i\ln\overline{y}_i)^2$
−1.8326	2.3627	–	–	–	–
−1.6094	2.474	–	–	–	–
−1.3863	2.6167	–	–	–	–
Σ = −4.8283	**7.4534**	**7.87041881**	**18.55014618**	**−11.93907083**	**2.54242333**

relations between logarithms: $\ln y = \ln\alpha + \ln x$ and $\ln y = \ln\alpha + x\ln b$ [12].

Table 9 presents the data required to calculate the regression coefficients and the correlation coefficient determining the degree of linearity of the linear dependence.

The regression coefficients of the linear dependence are

$$b = \frac{n\sum y_i x_i - \sum y_i \sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = 34.205;$$

$$\alpha = \frac{\sum y_i - b\sum x_i}{n} = 5.105.$$

The correlation coefficient determining the degree of linearity is

$$r = b\sqrt{\frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n\sum y_i^2 - \left(\sum y_i\right)^2}} = 0.999.$$

The linear dependence takes the form

$$\overline{HRC} = 5.105 + 34.205S.$$

Tables 10 and 11 present the data required to calculate the regression coefficients and the correlation coefficient determining the degree of linearity of the logarithmic expressions corresponding to power laws and exponential relations.

The regression factors of the power law and the correlation coefficient take the form

$$b = \frac{n\sum \ln y_i \ln x_i - \sum \ln y_i \sum \ln x_i}{n\sum \ln y x_i^2 - \left(\sum \ln x_i\right)^2} = 0.569;$$

$$\ln\alpha = \frac{\sum \ln y_i - b\sum \ln x_i}{n} = 3.4;$$

$$\alpha = \mathrm{e}^{\ln\alpha} = 29.964;$$

$$r = b\sqrt{\frac{n\sum \ln x_i^2 - \left(\sum \ln x_i\right)^2}{n\sum \ln y_i^2 - \left(\sum \ln y_i\right)^2}} = 0.997.$$

The power law takes the form $\overline{HRC} = 29.964S^{0.569}$.

The formulas for calculating the regression coefficients and the correlation coefficient determining the degree of linearity of the logarithmic expression are as follows

$$\ln b = \frac{n\sum \ln y_i x_i - \sum \ln y_i \sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = 2.823;$$

$$b = e^{\ln b} = 16.827; \quad \ln\alpha = \frac{\sum \ln y_i - \ln b\sum x_i}{n} = 1.91;$$

$$\alpha = \mathrm{e}^{\ln\alpha} = 6.753;$$

$$r = \ln b\sqrt{\frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n\sum \ln n y_i^2 - \left(\sum \ln y_i\right)^2}} = 0.999.$$

Table 11

x_i	$\ln\overline{y}_i$	x_i^2	$(\ln\overline{y}_i)^2$	$x_i\ln\overline{y}_i$	$(x_i+\ln\overline{y}_i)^2$
0.16	2.3627	–	–	–	–
0.2	2.474	–	–	–	–
0.25	2.6167	–	–	–	–
Σ = 0.61	**7.4534**	**0.1281**	**18.55014618**	**1.527007**	**21.73226018**

Table 12

Dependence	Empirical expression	Correlation coefficient r
1. Linear	$\overline{HRC} = 5.105 + 34.205S$	0.999
2. Power law	$\overline{HRC} = 29.964S^{0.569}$	0.997
3. Exponential	$\overline{HRC} = 6.753 \times 16.8357^S$	0.999

Table 13

Relation	S, mm/turn	$\overline{HRC}_{exp}$	$\overline{HRC}_{ca}$	Computational error, %	Mean error
1. $\overline{HRC} = 5.105 + 34.205S$	0.16	10.62	10.58	0.38	0.420
	0.20	11.87	11.95	0.67	
	0.25	13.69	13.66	0.21	
2. $\overline{HRC} = 29.964S^{0.569}$	0.16	10.62	10.56	0.56	0.690
	0.20	11.87	11.99	1.01	
	0.25	13.69	13.620	0.510	
3. $\overline{HRC} = 6.753 \times 16.8357^S$	0.16	10.62	10.610	0.090	0.076
	0.20	11.87	11.878	0.067	
	0.25	13.69	13.680	0.070	

The exponential relationship is $\overline{HRC} = 6.753 \times 16.8357^S$.

Table 12 presents the empirical formulas and correlation coefficients obtained by regression analysis. The similar values of the correlation coefficients do not permit the identification of the best relationship. Therefore, the most accurate relationship is selected by comparison. The comparison results are summarized in Table 13, which presents the computational error for each relation in Table 12.

The following relationship is the best

$$\overline{HRC} = 6.753 \times 16.8357^S. \quad (13)$$

On the basis of Eq. (13), we see the influence of the supply S and we may calculate the sample mean for the hardness of the machined surface. Hence, we may assess the generalized mean hardness by means of quantiles of the Student distribution $t_{1-p/2}$, using the expression

$$\overline{HRC} - \sqrt{\frac{s^2}{n}} t_{1-p/2} \le \alpha \le \overline{HRC} + \sqrt{\frac{s^2}{n}} t_{1-p/2}.$$

When $S = 0.2$ mm/turn, the sample mean of the hardness of the machined surface may be calculated from Eq. (13): $\overline{HRC} = 11.878$. From Table III in [3], we find that $t_{1-p/2} = 2.12$; $s^2 = 1.1$; and $n = 9$ (Table 5). Then the confidence interval of the generalized mean is $11.137 \le \alpha \le 12.62$.

With increase in the number of factors, the number of measurement series also increases. However, the statistical methods presented here for the analysis of the measurements and the order in which they are applied remain the same.

Thus, we have validated methods of statistically assessing the influence of specific factors on the qualitative characteristic of the experimental data, the generalized variance, which determines the accuracy of the experiment. This approach includes successively eliminating coarse measurement errors (outliers) using Grubbs criteria; confirmation that the measurements conform to a normal distribution; and variance analysis to determine the influence of the factors of interest on the qualitative characteristic of the experimental data, the generalized variance. The lack of such influence permits the use of statistical methods of assessing the influence of the factors of interest on the quantitative characteristic of the measurements, the generalized mean.

It has been shown that statistical assessment of the influence of the factor of interest on the generalized mean permits maximization of the accuracy of correlation and regression analysis.

Methods of increasing the accuracy of empirical models have been established, including assessment of the influence of specific factors on the generalized mean, correlation and regression analysis of the experimental results, and comparison of the accuracy of results obtained by regression analysis. On that basis, the accuracy of the empirical model may be maximized; the agreement between the calculation results and the experimental data exceeds 99.9%.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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